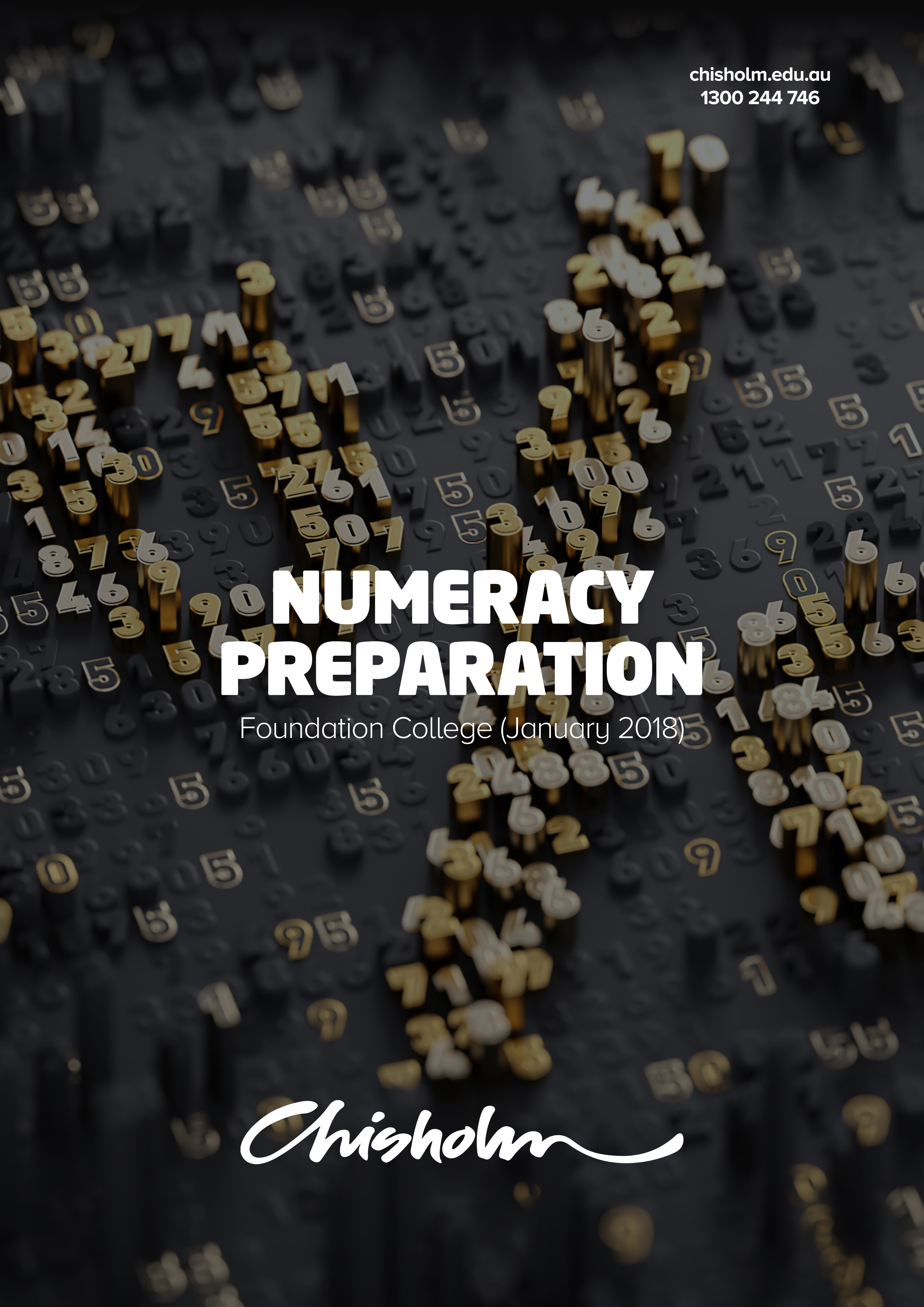

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NUMERACY PREPARATION

Foundation College (January 2018)

Chisholm

Maths Words

These are common words that you may find when you complete the numeracy assessment:

Addition	Subtraction
Symbol: + **Words:** Add Increase Plus More And Sum Total Together	**Symbol:** − **Words:** Take away Less Take from Reduce Take Fewer Minus Difference How many more
Multiplication	**Division**
Symbol: × **Words:** Multiply Times Product Multiplied by Doubled Groups of	**Symbol:** ÷ **Words:** Divided by Share Divide Group Divide into Share Equally Each

Order of Operations

B – Brackets

O – Of / Orders

D – Division

M – Multiplication

A – Addition

S – Subtraction

In maths, we must remember to always do the "groups of" and "sharing" parts first, before we add or subtract. We remember this with the acronym: BODMAS.

BODMAS reminds us to do *brackets* first,

Then any *multiplying or dividing*

And always do *adding and subtracting* last.

More Information can be found on:

https://www.mathsisfun.com/operation-order-bodmas.html

Rounding

Rounding means making a number simpler but keeping its value close to what it was.

Accuracy is often not required. In many cases an approximate answer is all that's needed.

When you use a calculator the answer can be more accurate than you need.

For Example: $1.21 is closest to $1.20

$2.49 is closest to $2.50

$3.06 is closest to $3.05

Decimals

Adding decimals

When adding decimal numbers, there are two important points to remember with the setting out:

The decimal points MUST align with (be underneath) each other.

When adding, the bottom number must have the same amount of numbers after the decimal point as the top number. This is done by adding zeros as required.

23.65		23.65
6.2	*Change to*	6.20
29.85		29.85

'Carrying' a number

$$\begin{array}{rcc}
 & 1 & \\
 & 1 & 8 \\
+ & & 4 \\
\hline
 & & 2
\end{array} \quad = \quad 12$$

All that is left to do is add together the numbers in the tens' column now. Calculation. It would look like this:

$$\begin{array}{rcc}
 & 1 & \\
 & 1 & 8 \\
+ & & 4 \\
\hline
 & 2 & 2
\end{array} \quad = \quad 22$$

More advanced addition problems with the addition of two digit plus two digit numbers with the need to carry are calculated by the very same method:

$$\begin{array}{rcc}
 & 4 & 7 \\
+ & 3 & 9
\end{array}$$

Carry the same way as above. Split up the number and write the units digit under the units' column beneath the answer bar, and write the tens' digit above the tens' column in the addition problem.

$$\begin{array}{rcc}
 & 1 & \\
 & 4 & 7 \\
+ & 3 & 9 \\
\hline
 & & 6
\end{array}$$

Add the tens' column together. There are three numbers in the tens' column that you must add together, 1 + 4 + 3.

$$\begin{array}{rcc}
 & 1 & \\
 & 4 & 8 \\
+ & 3 & 9 \\
\hline
\end{array}$$

	8 6

This information has been adapted from: "Building Strength With Numbers – Decimals. This is a free numeracy resource that can be obtained from: http://www.valbec.org.au/building-strength-with-numeracy/

Subtracting decimals

When subtracting decimal numbers, there are two important points to remember with the setting out:

The decimal points MUST align with (be underneath) each other.

When subtracting, the top number must have the same amount of numbers after the decimal point as the bottom number. This is done by adding zeros as required.

29.3	-	*Change to*	29.300
19.325			19.325
9.975			9.975

'Borrowing' a number

In the unit column, we are trying to subtract 9 from 6.

In order to do this, there is a need to ‘borrow’ from the 2 (the number in the tens’ column). Borrowing only has a few steps, but they have to be done in the right order for the problem to work out.

$$\begin{array}{rrr} & 2 & 6 \\ - & & 9 \\ \hline \end{array}$$

E.g. cross out the digit in the tens’ column. In our example, it is a 2.

$$\begin{array}{rrr} & \cancel{2} & 6 \\ - & & 9 \\ \hline \end{array}$$

Above the number you just crossed out, write the number that is one less than the one you crossed out. For example, if you cross out a 3, write 2; if you cross out a 2, write 1, and so on. We are going to write our new number in red as well.

$$\begin{array}{rrr} & 1 & \\ & \cancel{2} & 6 \\ - & & 9 \\ \hline \end{array}$$

In front of the units digit, write a 1. This is actually making it a two digit number so the 6 becomes 16, 2 becomes 12, and so on. For our example, it started with 2, so we put a 1 (in red) in front of the 6 to make it 16.

$$\begin{array}{rrr} & 1 & \\ & \cancel{2} & 16 \\ - & & 9 \\ \hline \end{array}$$

Start your subtraction over again, starting with the units’ column. Your new subtraction problem is 16 – 9. We know that 16 – 9 = 7, so we write 7 in the units column in the answer.

$$\begin{array}{rrr} & 1 & \\ & \cancel{2} & 16 \end{array}$$

$$\begin{array}{r r r} & & 9 \\ - & & \\ \hline & & 7 \end{array}$$

Now it is possible to continue with the rest of the subtraction, which is in the tens' column. Since there is only one digit in the tens' column, the number can be brought straight down into the tens' column of the answer.

$$\begin{array}{r r r} & 1 & \\ & \cancel{2} & 16 \\ - & \downarrow & 9 \\ \hline & 1 & 7 \end{array}$$

Subtracting two digit numbers

A two digit subtraction problem looks like this:

$$\begin{array}{r r r} & 7 & 4 \\ - & 3 & 8 \end{array}$$

In the units column we are trying to subtract 8 from 4 which cannot be done because 8 is bigger than 4. There is a need to go 'borrow' from the digit in the tens' column, which is the 7 in this example.

$$\begin{array}{r r r} & \cancel{7} & 4 \\ - & 3 & 8 \\ \hline & & \end{array}$$

Cross out the 7 and write 6 above the crossed out number.

$$\begin{array}{rcc} & 6 & \\ & \cancel{7} & 4 \\ - & 3 & 8 \\ \hline & & \end{array}$$

In front of the units' digit, write a 1. You are actually making it a 2-digit number when you do this, so 4 becomes 14:

$$\begin{array}{rcc} & 6 & \\ & \cancel{7} & 14 \\ - & 3 & 8 \\ \hline & & \end{array}$$

Start your subtraction over again, starting with the units column. Your new subtraction problem is 14 – 8. We know that 14 – 8 = 6, so we write 6 in the units column in the answer.

$$\begin{array}{rcc} & 6 & \\ & \cancel{7} & 14 \\ - & 3 & 8 \\ \hline & & 6 \end{array}$$

Continue with the rest of the subtraction, which is in the tens' column. This is where the step differs from before.
Previously, the number was just brought the top number straight down, because there was only one digit in the tens' column. Now, there are two digits in the tens' column, so we can subtract them. In the tens' column, we have 6 – 3, which equals 3. Therefore enter a 3 in the tens' column.

$$\begin{array}{rcc} & 6 & \\ & \cancel{7} & 14 \\ - & 3 & 8 \\ \hline & 3 & 6 \end{array}$$

This information has been adapted from: "Building Strength With Numbers – Decimals. This is a free numeracy resource that can be obtained from:
http://www.valbec.org.au/building-strength-with-numeracy/

Multiplying decimals

When multiplying decimals, it is easier to remove the decimal points, and then put the decimal point into the answer later.

When multiplying always count the number of decimal points that have been used in the calculation. Move the decimal point that many places to the left of the calculated answer

Step 1: Multiply the numbers as if they were whole numbers, ignoring the decimal point.

10.13 x 3.6 =

	1	0	1	3	x	
			3	6		
	6	0	7	8	+	1013 X 6
3	0	3	9	0		Move over one (1) space and multiply by three (3)
3	6	4	6	8		Add the two (2) responses together for the result

Step 2: Count the numbers after the decimal point in both numbers

3.6 has one number after the decimal place (.6)

10.13 has two numbers after the decimal place (.1 3)

Therefore you have to move the decimal place 3 places (1 + 2)

Step 3: Move the decimal point 3 places to the left in the result.

Answer: 36.468

Multiplying decimals by 10, 100, 1000

Numbers without decimal points are whole numbers. The decimal point is always after the whole number. For example, 6 can be written 6.0

To multiply

To multiply a decimal by 10, 100 or 1000 we need to look at the number of zeros in the multiplier and move the decimal point to the right by that many places.

6.23 x 10 = 62.3

In the calculation above there is one zero in the number 10 so the decimal point is moved one place to the right.

69.3653 x 1000 = 69365.3

Dividing decimals

Let's try this calculation:

11.88 ÷ 36 without a calculator

Step 1: The **divisor** in this example is 36. The **dividend** is 11.88

$$36\,\overline{)\,1\;1.\;8\;\;8}$$

Step 2: Divide the numbers as you would normally.

11.88 ÷ 36

dividend divisor

Here is a step by step process:

The calculation	The activity	What happens
$36\,\overline{)\,1\;\;1.\;\;8\;\;\;8}$	1 ÷ 36 = 0	The first digit of the dividend (1) is divided by the divisor.
$\begin{array}{r} 0 \\ 36\,\overline{)\,1\;\;1.\;\;8\;\;\;8} \end{array}$		The whole number result is placed at the top. Any remainders are ignored at this point.
$\begin{array}{r} 0 \\ 36\,\overline{)\,1\;\;1.\;\;8\;\;\;8} \end{array}$	36 × 0 = 0	The answer from the first operation is **multiplied** by the

0		divisor. The result is placed under the number divided into.
0 36) 1 1. 8 8 0 1	$1 - 0 = 1$	Now we **subtract** the bottom number from the top number.
0 36) 1 1. 8 8 0 1 1	0 36) 1 1. 8 8 0 1 0	Bring down the next digit of the dividend.
0 0 36) 1 1. 8 8 0 1 1	$11 \div 36 = 0$	The whole number result is placed at the top. Any remainders are ignored at this point.
0 0 36) 1 1. 8 8 0 1 1	$36 \times 00 = 0$	We can now **subtract** the bottom number from the top number.
0 0 36) 1 1. 8 8 0 1 1 8		Bring down the next digit of the dividend.
0 0 36) 1 1. 8 8 0 1 1 8	$118 \div 36 = 3$ remainder	**Divide** this number by the divisor.

0 0 3 36 | 1 1. 8 8 0 1 1 8		The whole number result is placed at the top. Any remainders are ignored at this point.
0 0 3 36 | 1 1. 8 8 0 1 1 8 1 0 8	36 x 3 = 108	The answer from the above operation is **multiplied** by the divisor. The result is placed under the last number divided into.
0 0 3 36 | 1 1. 8 8 0 1 1 8 1 0 8 1 0	118 - 108 = 10	Now we **subtract** the bottom number from the top number.
0 0 3 36 | 1 1 8 8 0 1 1 8 1 0 8 1 0 8		Bring down the next digit of the dividend.
0 0 3 36 | 1 1. 8 8 0 1 1 8 1 0 8 1 0 8	108 ÷ 36 = 3	**Divide** this number by the divisor.

0 0 3 3 36 | 1 1. 8 8 0 1 1 8 1 0 8 1 0 8		The whole number result is placed at the top. Any remainders are ignored at this point.

0 0 3 3 36 | 1 1. 8 8 0 1 1 8 1 0 8 1 0 8 1 0 8	36 x 3 = 108	The answer from the above operation is **multiplied** by the divisor. The result is placed under the last number divided into.
0 0. 3 3 36 | 1 1. 8 8 0 1 1 8 1 0 8 1 0 8 1 0 8 0	108 – 108 = 0	Now we **subtract** the bottom number from the top number.

Step 3: Put the decimal point in the quotient (answer) directly above the decimal point in the dividend.

0 0. 3 3 36 | 1 1. 8 8	**Answer:** 0.33

10.332 ÷ 36 =

This can also be written as $36\overline{\big)10.332}$

Divide the numbers as you would normally.

	0	0	2.	8	7
36	1	0	3.	3	2
	0				
	1	0	3		
		7	2		
		3	1	3	
		2	8	8	
			2	5	2
			2	5	2
					0

Answer: 2.87

Dividing decimals by 10, 100, 1000

To divide a decimal by 10, 100 or 1000, the decimal point moves *to the left* by the same number of zeros.

$$36.2 \div 10 = 3.62$$

$$69.23 \div 1000 = 0.06923$$

At times there are not enough numbers to move the decimal point past so we need to add one or more zeros in front of the first number. This time your calculator would show a zero in front of the decimal point as well as adding one after the decimal point.

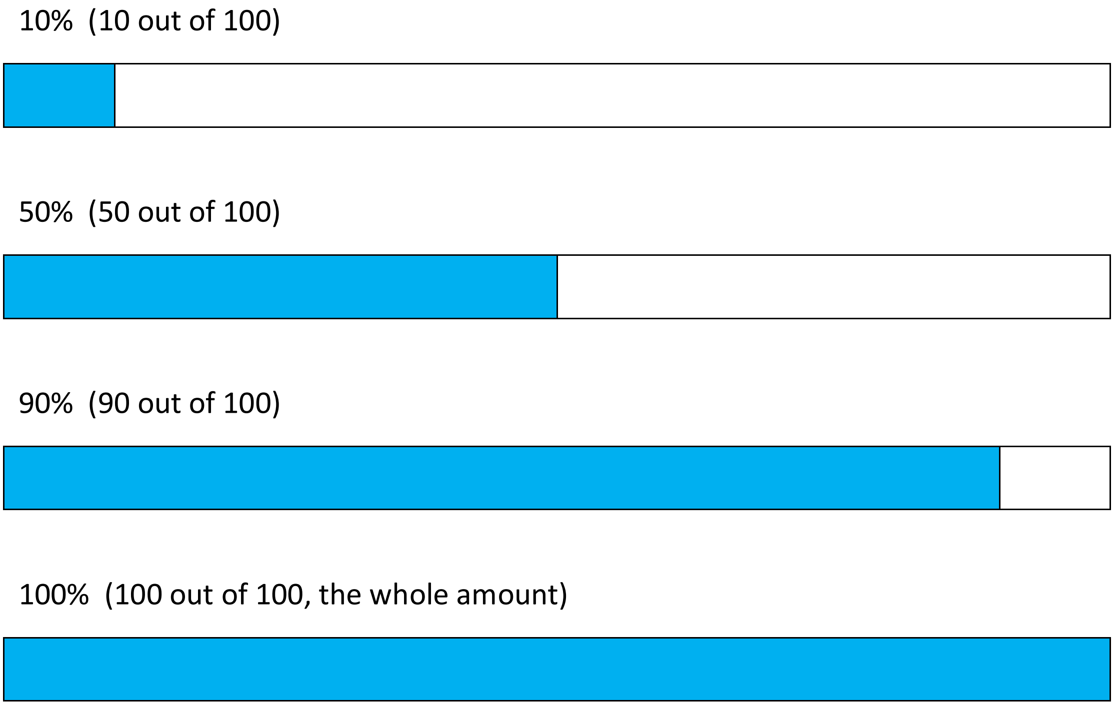
This information has been adapted from: "Building Strength With Numbers – Decimals. This is a free numeracy resource that can be obtained from:
http://www.valbec.org.au/building-strength-with-numeracy/

Percentages

The % symbol tells us that we are dealing with a portion of 100.

Percent means 'out of one hundred'.

The percentage of an amount is the number of small parts we have of the total of 100 parts.



1% means 1 out of 100 and is the same as $\frac{1}{100}$, in other words to find 1% of an amount, divide by 100. For example: 1% of \$500 is \$500 ÷ 100 = \$5.

Calculating percentages

To calculate the *percentage of* an amount:

1. **Divide the percentage by 100**
2. **Multiply this by the amount**

For example: **What is 30% of $200?**

30 ÷ 100 x $200 = $60

Here are some examples:

Percentage	Amount of Money	Sum
30%	$200	30 ÷ 100 x 200 =
45%	$200	45 ÷ 100 x 200 =
35%	$200	35 ÷ 100 x 200 =
60%	$200	60 ÷ 100 x 200 =
50%	$500	50 ÷ 100 x 500 =
25%	$500	25 ÷ 100 x 500 =
75%	$500	75 ÷ 100 x 500 =

50% = one-half
(divide by 2)

25% = one-quarter
(divide by 4)

10% = one-tenth
(divide by 10)

Percentage Change

Sometimes we need to work out what the percentage change (increase or decrease) is.

$$\textbf{Percentage Change} = \frac{\textbf{amount } \boldsymbol{of\ increase\ or\ decrease}}{\boldsymbol{Original\ price}}\ \boldsymbol{x}\ \mathbf{100}$$

For example a shop has reduced the cost of a T-Shirt from $20 to $10

$$\text{Percentage Change} = \frac{10}{20}\ x\ 100$$

= 0.5 x 100

= 50%

Percentages as 'Parts of a Total'

Sometimes you may have something that is a part of a larger amount and you need to know what percentage it represents.

1. **Divide the smaller part by the whole amount. You will end up with a decimal.**
2. **Multiply the decimal by 100 to get the percent.**

For example: I spend $20 per week on my phone and my total income is $400

1. 20 ÷ 400 = 0.05
2. 0.05 x 100 = 5%

I spent 5% of my income on my phone.

Here are some examples:

Example 1:

10% of 456 is 45.6

10% of \$14.25 is \$1.43 (rounding up)

Example 2:

The price of a spanner is \$6.

Today everything is 30% off.

What will today's price be?

10% is \$0.60

So 30% is 3 x 0.60 = 1.80

Price will be \$6 - \$1.80 = \$4.20

Example 3:

For example: If Jayden earns $250 per week at the local fish and chip shop and he receives a 2% pay increase, how much will he now earn per week?

1% of $250 is $2.50

2% of $250 is $5.00 (twice as much)

So, he will now earn: $255 per week

Final Note:

If 10% is easy to find, you can find 20% by doubling it, or 5% by halving it, or 30% by multiplying it by 3. You could find 15% by adding 10% and 5%.

Source: Building Strength With Numeracy 2013 VALBEC www.valbec.org.au (adapted by: SandersD) http://www.valbec.org.au/building-strength-with-numeracy/
http://www.valbec.org.au/building-strength-with-numeracy/

Fractions

Fractions are an equal part of a whole number. Fractions have a numerator (number on top) and a denominator (number on the bottom).

This is how to write a fraction:

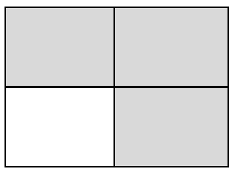
$\frac{3}{4}$

Numerator: number of parts taken

Fraction bar

Denominator: number of equal parts the whole is broken into

The drawing shows $\frac{3}{4}$

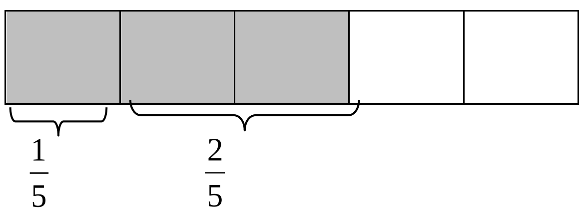


Adding and Subtracting Fractions

> **To add or subtract fractions, they must have the same denominator.**

If they already have the same denominator, just add or subtract the numerators

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$



Multiplying Fractions

When multiplying fractions the following words are used: times, of, lots of.

The rule for multiplying fractions is:

- **Place the whole number over 1.**
- **Multiply the numerators and then multiply the denominators**

Example:

$$\frac{2}{3} \times \frac{4}{5} = \qquad \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

More Information and activities to practise can be found on:

https://www.mathsisfun.com/fractions.html

Converting Fractions to Percentages

To convert a fraction to a percentage we must first convert the fraction to the decimal.

1. **Divide the numerator by the denominator to find the decimal**
2. **Multiply the decimal by 100 to get the percentage.**

More Information and activities to practise can be found on:

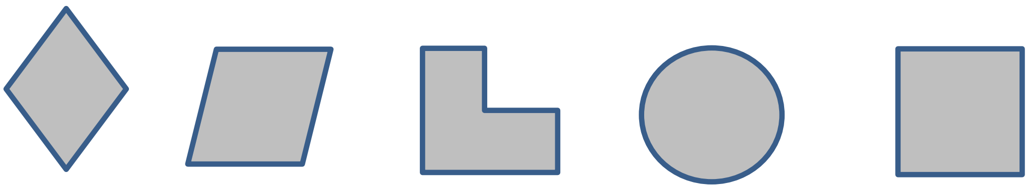
https://www.mathsisfun.com/fractions.html

Shapes

2-D Shapes

In numeracy we call a shape that is *flat*:   two dimensional.

If it is *flat*, it has *no* height or depth. So, it has only 2 dimensions: <u>Length</u> and <u>width</u>!   It's 2-D!!



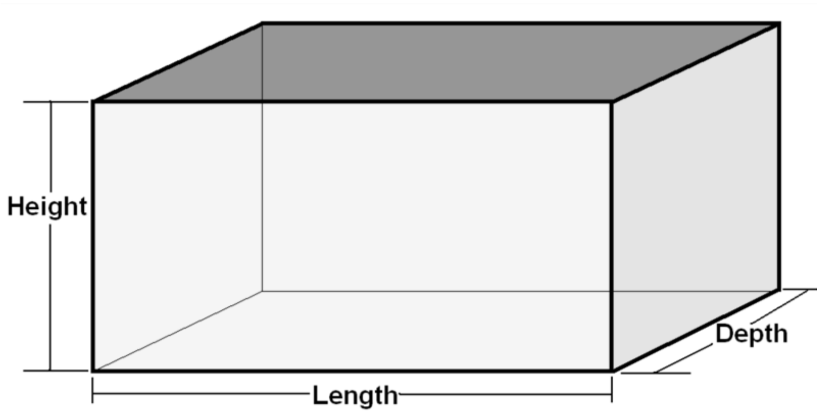
Here are some examples of 2D shapes

Search the internet for more 2D shapes: Use: *<u>2D shapes</u>* as your key words!

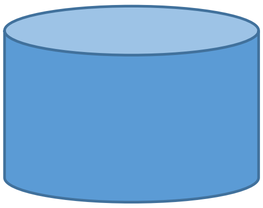
A polygon is any 2-dimensional shape formed with straight lines. Triangles, quadrilaterals, pentagons, and hexagons are all examples of polygons. The name tells you how many sides the shape has.

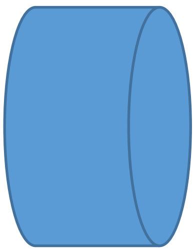
3-D Shapes

Three dimensional shapes (3-D) have 3 dimensions: **length, width** and **depth**.



We live in a 3-D world. Everything in our world has 3 dimensions.





Search the internet for more 3d shapes: Use: *<u>3D shapes</u>* as your key words!

Shapes have: Faces, Vertices and Edges

A vertex (plural: vertices) is a point where two or more lines meet. It is a Corner.

An edge is a line segment that join two vertices.

A face is any of the individual surfaces of a solid object.

Euler's Formula

For many solid shapes the Number of Faces plus the Number of Vertices minus the Number of Edges always equals 2

This can be written: $F + V - E = 2$

More Information and activities to practise can be found on:

https://www.bbc.co.uk/education/guides/zj76fg8/revision

Measurement

Time

To convert 12-hour time to 24-hour time use this method:

From 1:00 PM to 11:59 PM you add 12 hours, and from 12:00 AM (midnight) to 12:59 AM you subtract 12 hours.

Search the internet for a table to show you 12 hour and 24 hour time: Use: *24 hour time* as your key words!

This link will take you to an online converter:

www.onlineconversion.com/date_12-24_hour.htm

Metric Units

The commonly used metric units of length include:

- kilometres (Km)
- metres (m)
- centimetres (cm)
- millimetres (mm)

The commonly used metric units of mass include:

- gram (g)
- kilogram (kg)
- tonne (t)
- milligram (mg)

The commonly used metric units of capacity include:

- litre (l)
- millilitre (ml)
- cubic centimetre (CC)

The table shows some of the most common units and their equivalents.

Length	1 km = 1,000 m	1 m = 100 cm	1 cm = 10 mm
Weight	1 tonne = 1000 kg	1 kg = 1,000 g	1 g = 1,000 mg
Capacity	1 l = 100 cl	1 cl = 10 ml	1 l = 1,000 ml

Converting larger units to smaller units

To convert a larger unit to a smaller unit (eg m to cm), first check the number of smaller units needed to make one larger unit. Then, multiply that number by the number of larger units.

Converting smaller units to larger units

To convert a smaller unit to a larger unit (eg cm to m), divide it by the number of smaller units which are needed to make one larger unit.

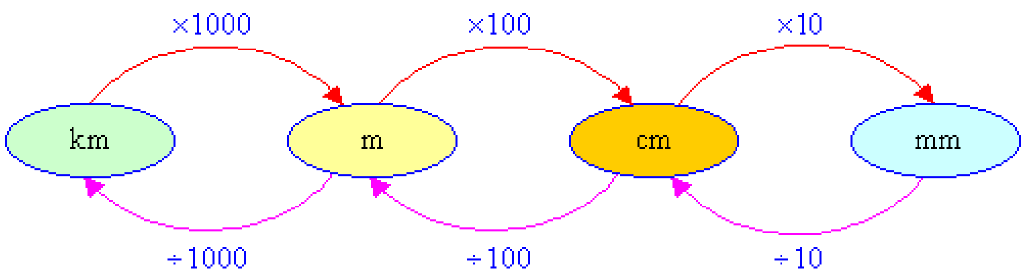
Converting between metric units of length is made easy because we only ever need to multiply or divide by **10, 100, 1000,** etc. to change to different units. This is done by simply moving the decimal point in the value being converted.

This shows the key information you need to convert between units of length.

10mm = 1cm

100cm = 1m

1000m = 1km



Examples: Converting Length

mm ÷ 10 = cm 25 mm ÷10 = 2.5 cm	cm x 10 = mm 4 cm x 10 = 40 mm
cm ÷100 = m 400 cm ÷100 = 4 m	m x 100 = cm 3.6 m x 100 = 360 cm
m ÷ 1000 = km 4800 m ÷ 1000 = 4.8 km	km x 1000 =m 6.6 km x 1000 = 6600 m

__Converting between Grams and Kilograms__

1000g = 1kg

g ÷ 1000 = kg 5100 g ÷ 1000 = 5.1 kg	kg x 1000 =g 6.5 kg x 1000 = 6500 g

__Converting between Millilitres and Litres__

1000mL = 1L

mL ÷ 1000 = L 2000 mL ÷ 1000 = 2 L	L x 1000 =mL 1.5 L x 1000 = 1500 mL

Here are some examples:

Km ⟷ 1000 ⟷ m ⟷ 100 ⟷ cm ⟷ 10 ⟷ mm

Move the decimal point in the direction of the unit you want to convert to.

Example 1.

175 *mm* = *cm*

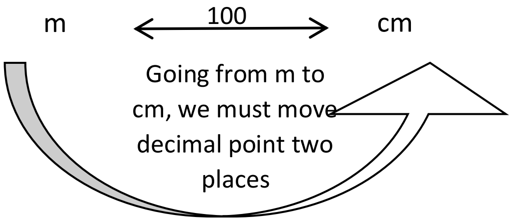
cm $\xleftrightarrow{10}$ mm

Going from mm to cm, we must move decimal point one place

175 *mm* = 17.5 *cm*

Example 2.

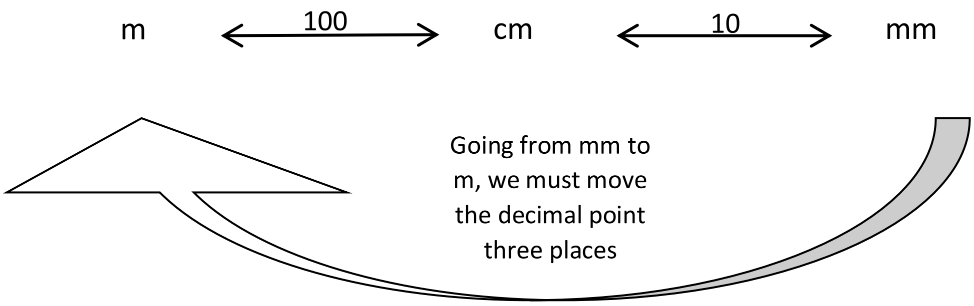
2.3 *m* = *cm*



2.3 *m* = 230 *cm*

Example 3.

2075 *mm* = *cm*



2 0 7 5 *mm* = 2.075 *cm*

(Created R McKenzie – Foundation College 2015)

More Information and activities to practise can be found on:

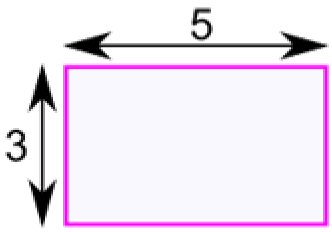
https://www.bbc.co.uk/education/guides/zthsgk7/revision/3

Area and perimeter

Perimeter is the total distance around a shape which has straight sides.

Perimeter – Add all sides of the shape

For example the perimeter of this shape is:



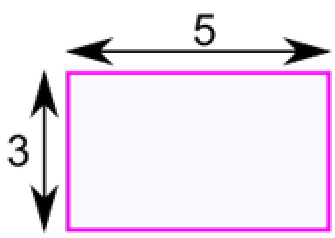
5 + 3 + 5 + 3 = 16

Area is number of unit squares that can be contained within a shape

Area – The formula is:

Area = w × h
w = width
h = height

For example the area of this shape is:



The width is 5, and the height is 3, so we know **w = 5** and **h = 3**: Area = 5 × 3 = **15**

Capacity and Volume

We use the term capacity when talking about the measure of how much space there is available to hold something.

For example the capacity of:

a jug
a teacup or mug
a food container
a petrol tank

Capacity is the amount a container can hold.

Volume is something slightly different.

Here's an example:

A jug of juice has a capacity of 500 ml.

The **volume** of juice is how much is actually in the jug

The **volume** of juice needed to fill the jug is 500 ml.

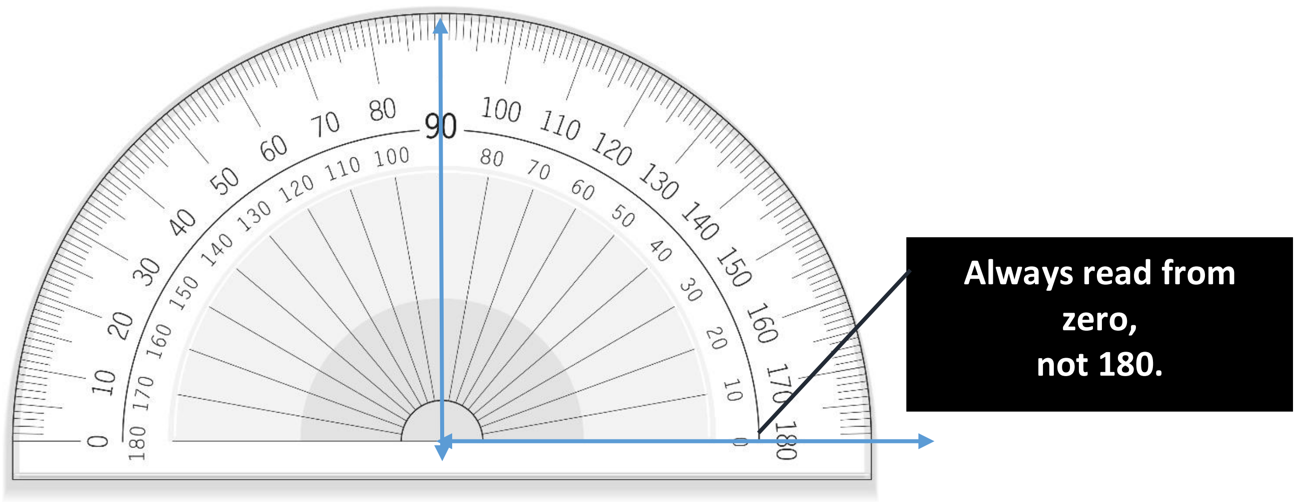
Volume is a measure of the space taken up by something.

This website provides many examples showing how to calculate volume of various shapes: https://www.helpingwithmath.com/by_subject/geometry/geo_volume.htm

Angles - Measuring angles between 0© and 180©

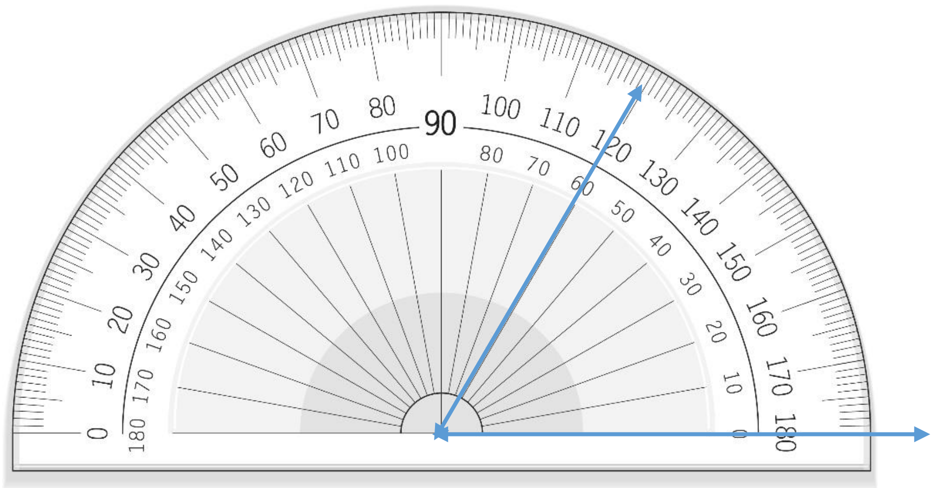
How many degrees is each angle?

This is 90 degrees



(shutterstock_146377604)

This is 60 degrees

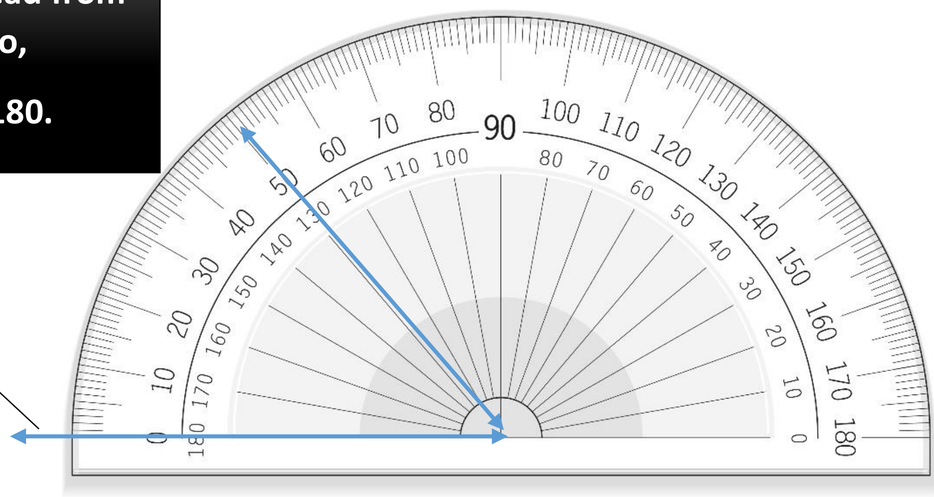


(shutterstock_146377604)

This is 50 degrees

Always read from zero,

not 180.



(shutterstock_146377604)

Mean

The "**mean**" is the "average" - you add up all the numbers and then divide by the number of numbers.

Calculate the mean by adding all the numbers in the set of data and then dividing the total by how many numbers you have.

Example:

In this data set we have six numbers
7 45 33 4 12 25

Added together = 126
Then divide by 6 (because there are 6 numbers) = 21

The mean of these numbers is 21

Median

The "**median**" is the "middle" value in the list of numbers.

Calculate the median by ordering the numbers from smallest to largest and identifying the middle number

Example:

Here is my original data collected about house numbers:
11 77 18 43 33 59 27

Now I must order the data from smallest to largest
11 18 27 <u>33</u> 43 59 77

The number in the middle is 33.

Therefore, the median is 33.

BUT….What if there is an even number of numbers?

For example, here is my data collected about house numbers.
11 77 18 43 33 59

Now I must order the data from smallest to largest
11 18 <u>33 43</u> 59 77

33 and 43 are both the middle numbers.

To calculate the median we need to take the mean of these two values (add them together and divide by 2).

33 + 43 = 76
76 ÷ 2 = 38

The median is 38

Mode

The **mode** is the number that is repeated the most

To calculate the mode we need to count how many times each number occurs.

For example, here is my data collected about ages of students.

17	17	16	18	19	18	19	17	21	23

17 occurs three times in this set of data, more than any other number.
The mode is 17.

*Note – sometimes there will be more than one mode.

Range

The **range** is the difference between the highest and lowest number in the data set

To calculate the range we identify the largest number in the data set and subtract the smallest number in the data set.

For example, here is my data collected about ages of students in our CGEA class.

17	17	16	18	19	18	19	17	21	23

The largest number in the data set is 23
The smallest number in the data set is 16

23 – 16 = 7

The range of this data is 7

Ratio and Rates

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:).

If we want to write the ratio of 8 and 12. We can write this as 8:12 and we say the ratio is *eight to twelve*.

Example:

In her school bag Jane has 4 markers and 7 books

What is the ratio of books to markers - Two ways of writing the ratio are 7 to 4, and 7:4.

What is the ratio of markers to the total number of items in the bag?
There are 4 markers, and 4 + 7 = 11 items in total. The answer can be expressed as 4/11, 4 to 11 or 4:11

Remember to be careful! Order matters! A ratio of 4:11 is not the same as a ratio of 11:4

Simplifying Ratios

Problems are often made easier by putting the ration into their simplest terms.
The ratio 1 : 3,   3 : 7 and 9 : 4 are in their simplest terms because there is no number which will divide exactly into both sides.

The ratio 8 : 6 is not in its simplest terms because 2 will divide into both sides to give 4 : 3 which is the same as 8 : 6.

Another example

Put the ratio 72 : 84 in its simplest terms.
12 divides into 72 and 84, hence

72 : 84 is the same as

72 ÷ 12 : 84 ÷ 12. That is 72 : 84 is the same as 6 : 7

Measures of Rate

A **rate** is a ratio that expresses how long it takes to do something, such as travelling a certain distance.

Examples:
To walk 3 kilometres in one hour is to walk at the rate of 3 km/h.

If a car travels 10 km on 1 litre of fuel we say its fuel consumption is 10 km per litre of petrol. This is the rate at which the car consumes fuel.

The flow of water from a tap is usually measured in litres per minute. This is the rate of flow of the water.

Indices, Powers and Roots

Indices are a useful way of expressing large numbers in a simple way.

Powers, are a way of indicating that a quantity is to be multiplied by itself some number of times. In the expression 2^5, 2 is called the base and 5 is called the exponent, or power. 2^5 is shorthand for "multiply five twos together": $2^5 = 2\times2\times2\times2\times2 = 32$.

The **root** of a number is another number, which when multiplied equals the given number. For example the second root of 9 is 3, because 3x3 = 9. The second root is usually called the square root

Here is some helpful language:

1. a x b can be written as ab

2. 5 x m can be written as 5m

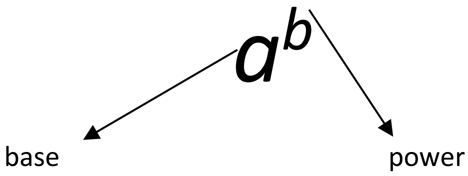
3. P x p is the same as pp which is the same as p^2

4. 3 x 3 is the same as writing 3^2

5. 3^2 is read as "3 to the power of 2" or "3 squared."

6. n^3 is read as "n to the power of 3" or "n cubed."

7.



8. 5^4 means the base (5) is multiplied by itself 4 times
$5^4 = 5 \times 5 \times 5 \times 5$

9. t^7 means the base (t) is multiplied by itself 7 times.
$t^7 = t \times t \times t \times t \times t \times t \times t$

10. $8^1 = 8$ which also means, of course, that $8 = 8^1$
$a^1 = a$ which also means, of course, that $a = a^1$

Algebra

Algebra is about finding the unknown or putting real life variables into equations and then solving them

LIKE terms are terms with have EXACTLY the same letters. It doesn't matter if they are in a different order and it doesn't matter if the coefficients (numbers) are different.

Examples:

2*e* and -5*e* and -2*e* are **LIKE** terms

-7*ab* and 12*ab* are **LIKE** terms

5*y* and 5*z* and 5*yz* are **UNLIKE** TERMS

2*abc* and 2*bcd* and -2*ab* are **UNLIKE** TERMS

Adding and subtracting like terms

In algebra, expressions can often be simplified. We can add or subtract LIKE terms.

$a + a + a + b + b$

$= 3a + 2b$ (can't go any further because 3a and 2b are UNLIKE terms.)

Changing the Subject of Formula

Formula means the relationship between two or more variables

Subject of a formula means the variable on its own, usually on the left hand side of the equals sign.

Changing the subject of formula means to rearrange the formula so that a different variable is on its own.

Imagine a formula to be like an onion, to re-arrange it you need to remove each layer from the variable you want to be left as the subject.

- **The order in which you do this is important. Generally, start at the outside of the formula and work your way in.**
- **Just like solving any other equation, when you move anything across the equals sign you must invert its operation.**

For a visual example log onto the Internet and watch this video by Sarah Chammings https://www.youtube.com/watch?v=cbKc_qilgzA (or go to Youtube and type 'Sarah Chammings How to change the subject of a formula'

Example 1, make x the subject of the formula

$y = x + 5$ Here, $+5$ is on the outside of the equation.

To 'peel this away' I need to undo it by inverting its operation

$y \cancel{-5} = x + 5 \cancel{-5}$ I will subtract 5 from both sides

$y - 5 = x$ Now x is on its own and therefore the subject of the formula

Example 2 Make x the subject of the formula

$y = 2x - a$ $-a$ is on the outside. Let's add a to both sides

$y + a = 2x$ x is attached to the 2 with a multiplication.

Lets invert that operation and divide both sides by 2

$\frac{y+a}{2} = x$ Now x is the subject of the formula

Example 3 Make x the subject of the formula

$y = \frac{x}{5}$ The right hand side is divided by 5

So, lets multiply both sides by 5

$5y = x$ Now x is the subject of the formula

Expanding and Simplifying and Substitution into Equations

Expanding means that you need to remove or expand the brackets

Examples

Expand these expressions:

$3\ (x - 6)$ Remember there is an invisible multiplication sign here. $\times$ $3\ (x - 6)$ $= 3x - 18$	$-3(m + 2)$ When there is a NEGATIVE number outside the bracket, the signs INSIDE the bracket changes. $= -3m - 6$

Here are more guidelines for algebra:

LAW 1

When multiplying terms with the SAME base, ADD the indices.

$$a^m \times a^n = a^{m+n}$$

Example 1

$a^3 \text{ x } a^2 = a \text{ x } a \text{ x } a \text{ x } a \text{ x } a = a^5$

Example 2

$4a^3 \text{ x } 5a^2 = 4 \text{ x } a \text{ x } a \text{ x } a \text{ x } 5 \text{ x } a \text{ x } a \text{ x } a = 20a^5$

Example 3

$\underline{10}^4 \text{ x } \underline{10}^2 = \underline{10}^6$

(base is 10   base is 10   base is still 10)

LAW 2

When **dividing** terms with the **SAME** base, **subtract** the indices

$$a^m \div a^n = a^{m-n}$$

Example 1

$$x^5 \div x^3 = \frac{x^5}{x^3} = \frac{\cancel{x} \times \cancel{x} \times \cancel{x} \times x \times x}{\cancel{x} \times \cancel{x} \times \cancel{x}} = x^2$$

Can you see a shortcut from here to here?

Example 2

(a) $h^6 \div h = \frac{h^6}{h} = \frac{h^6}{h^1} = h^5$

Example 3

(b) $\frac{10b^4}{2b} = \frac{10b^4}{2b^1} = \frac{10}{2}\frac{b^4}{b^1} = 5b^3$

Example 4

(c) $\frac{5^6 m^5 n^2}{5^4 m^2 n} = \frac{5^6}{5^4} \frac{m^5}{m^2} \frac{n^2}{n} = 5^2 m^3 n^1 = 5^2 m^3 n$

Example 5

(d) $\frac{6^5}{3^4} =$ index law 2 does not apply because the base numbers (6 and 3) are different

LAW 3

When there is one index raised to another index, multiply the two index numbers.

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Example 1 $(2^3)^2 = 2^3 \times 2^3 = 2^6$

Can you see a shortcut from here to here?

Example 2 $(a^4)^3 = a^4 \times a^4 \times a^4 = a^{12}$

Can you see a shortcut from here to here?

Example 3

$(2b^3)^3 = (2^1 b^3)^3 = 2^3 b^9$

Example 4

(a) $(4p^4)^2 = 4^2 p^8 = 16p^8$

<u>Every</u> term (in this case, the 4 and the p^4) inside the bracket is affected by the index outside the bracket.

Example 5

$(m^4 n^6)^3 = m^{12} n^{18}$

Example 6

$\left(\frac{3^2 a^3}{5b^2}\right)^3 = \frac{3^6 a^9}{5^3 b^6}$

Example 7

(d) $5(e^2)^5 = 5e^{10}$

LAW 4

Any term raised to the power of zero is equal to 1.

$a^0 = 1$

Example 1

$$\frac{3^2}{3^2} = 3^{2-2} = 3^0$$

These two answers must be equal to each other.

Therefore, it follows that:

$3^0 = 1$

$$\frac{3^2}{3^2} = \frac{3\times3}{3\times3} = \frac{1\times1}{1\times1} = \frac{1}{1} = 1$$

Example 2

$$\frac{y^4}{y^4} = y^{4-4} = y^0$$

These two answers must be equal to each other.

Therefore, it follows that:

$Y^0 = 1$

$$\frac{y^4}{y^4} = \frac{y\times y\times y\times y}{y\times y\times y\times y} = \frac{1\times1\times1\times1}{1\times1\times1\times1} = \frac{1}{1} = 1$$

Example 3

(a) $2b^0 = 2 \times b^0 = 2 \times 1 = 2$ (Here, only the b is raised to the power of zero)

(b) $-12m^0 = -12 \times m^0 = -12 \times 1 = -12$

(c) $(5h)^0 = 1$

(d) $5(h)^0 = 5 \times 1 = 5$

(e) $4a^0 - 1 = 4 \times a^0 - 1 = 4 \times 1 - 1 = 4 - 1 = 3$

(f) $(7d^2k^5)^0 + 1 = 1 + 1 = 2$

(g) $7(d^2k^5)^0 + 1 = 7 \times (d^2k^5)^0 + 1 = 7 \times 1 + 1 = 7 + 1 = 8$

LAW 5

If a negative power is on the <u>numerator</u>, put it to the <u>denominator</u> and change it to positive.

$$a^{-n} = \frac{1}{a^n}$$

Also, if a negative power is on the <u>denominator</u>, put it to the <u>numerator</u> and change it to positive.

$$\frac{1}{b^{-n}} = b^n$$

Example 1

$$\frac{2^3}{2^5} = 2^{-2}$$

These two answers must be equal to each other.

Therefore, it follows that:

$$2^{-2} = \frac{1}{2^2}$$

$$\frac{2^3}{2^5} = \frac{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1}{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2}$$

Example 2

These two answers must be equal to each other.

Therefore, it follows that:

$$\frac{b^4}{b^7} = b^{-3}$$

$$b^{-3} = \frac{1}{b^3}$$

$$\frac{b^4}{b^7} = \frac{\cancel{b}^1 \times \cancel{b}^1 \times \cancel{b}^1 \times \cancel{b}^1}{\cancel{b}^1 \times \cancel{b}^1 \times \cancel{b}^1 \times \cancel{b}^1 \times b \times b \times b} = \frac{1}{b \times b \times b} = \frac{1}{b^3}$$

* It is a mathematical convention to give answers with **positive** indices wherever possible.

Example 3

Write the following with positive indices and simplify wherever possible.

(a) $2d^{-4} = 2 \times d^{-4} = \dfrac{2}{d^4}$

(b) $\dfrac{3^{-3}}{c^{-2}} = \dfrac{c^2}{3^3} = \dfrac{c^2}{9}$

(c) $\dfrac{1}{8^{-3}} = 8^3$

(d) $3^{-2}a^2b^{-1} = \dfrac{a^2}{3^2b^1} = \dfrac{a^2}{9b}$

LAW 6

A number raised to a fraction index $(\frac{1}{n})$ represents the n^{th} root of that number.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Remember square roots?

$\sqrt{9} = 3$ and $\sqrt{25} = 5$ etc..

Another way of writing square roots is with a fraction index

Like this: $9^{\frac{1}{2}} = \sqrt{9} = 3$ and $25^{\frac{1}{2}} = \sqrt{25} = 5$

Example 1

$16^{\frac{1}{2}} = \sqrt{16} = 4$

Example 2

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ (because 3×3×3=27)

Example 3

$1^{\frac{1}{2}} = 1$

Example 4

$32^{\frac{1}{5}} = \sqrt[5]{32} = 2$ (because 2×2×2×2×2=32)

(Source: Original curriculum developed by Robyn McKenzie – Foundation College – April 2015)

Useful Websites to practice your Literacy skills. These are all free access:

Highly recommended. This one contains interactive activities so you can check your answers:

https://www.bbc.co.uk/education/subjects/zqhs34j

https://www.mathsisfun.com/

https://www.helpingwithmath.com

For maths and numeracy reference:

http://www.schoolatoz.nsw.edu.au/homework-and-study/mathematics/help-sheets

https://www.khanacademy.org/math

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